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ET DE LA RECHERCHE EN SYSTÉMATIQUE APPLIQUÉE

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Predictive control of the biomass production of the photoheterotrophic compartment. Simulator based on the new first principles model

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Abbreviations :
GPS : Global Purpose Station
MV : Manipulated Variable
PFC : Predictive Functional Control
PBR : PhotoBioReactor

N.B. : The notations are those of TN 45.1.

The inputs of the process are the light flux, F_0 , and the flow rate, Q, which is assumed to be identical to the level 1 flow rate set point, CQ1, at any moment.

The control is composed of 2 levels. The upper level, level 2, computes the level 1 production and flow rate set points, CProd1 and CQ1, in function of the respective level 2 set points, CP2 and CQ2, taking into account the maximum and minimum concentration constraints and the flow variation dq (±10 % of CQ2). The lower level, level 1, computes the light energy flux, F_0, function of the set point CProd1 and taking into account the disturbance Q.
The code of the control is written with C language. It is designed in such a way that it can be moved from the simulator into the GPS structure (present or future) with the less possible modifications.
At the meeting ESA/LGCB/ADERSA on May 12th 1999, it was decided to give an official name to a few software's :

name	software
LSPC	Light Spirulina Production Control
LRPC	Light Rhodobacter Production Control
SBQ	Spirulina Biomass Quality
MCS	Melissa Control Software (Level 2 of the hierarchical control)

So, according to this decision, the main function realizing the control of the Rhodobacter production by action on the light intensity is called now *lrpc* (source file in Annex 5).

2. CONTROL STRUCTURE

From the point of view of the biomass production control by action on light, the two photobioreactor with Spirulina and Rhodobacter are very similar. So the control structure is identical in the 2 cases. Furthermore, this attitude reduces maintenance costs.
Full details have been given in TN 24.1 about the levels 1 and 2 of the control. Here are a few recalls.

Level 1 control

This controller is built on the principles of PFC (Predictive Functional Control) :

- internal model ;
- reference trajectory to specify the future closed loop behaviour ;
- manipulated variable structuration ;
- modelling error extrapolation.

A scenario strategy is used to adapt the classical PFC control (which is convenient for linear process) to the present process which is highly non linear : an iterative input protocol is applied to the model from which a prediction of the model output is inferred (annex 1).

Level 2 control

This level checks the compatibility flow rate and production set points, fixed by the operator (or a upper level), with regard to variation flow rate and concentration constraints.

Algorithm : at each period of time, the following computations are made :

1 - maximum and minimum flow rate function of the level 2 flow rate set point cq2 :

$$q\_max = cq2\ (1+ dq)$$
$$q\_min = cq2\ (1 - dq)$$

2 - maximum and minimum production function of the absolute constraints on C_X :

$$p\_max = q\_max * C_{X}\_max$$
$$p\_min = q\_min * C_{X}\_min$$

3 - level 1 (or feasible) production set point :

$$cp1 = \max [p\_min, \min (p\text{-}max, cp2]$$

4 - level 1 (or feasible) flow rate set point :

$$\text{if}\left(\frac{cp1}{C_x\_max} > cq2\right) \text{ then}$$

$$cq1 = \min\left(q\_max, \frac{cp1}{C_x\_max}\right)$$

$$\text{else if}\left(\frac{cp1}{C_x\_min} < cq2\right) \text{ then}$$

$$cq1 = \max\left(q\_min, \frac{cp1}{C_x\_min}\right)$$

else
$cq1 = cq2$

Notations

C_X	Total biomass concentration (kg/m^3 or g/l)
$\mathcal{E}_I$	Mean efficient intensity (W/m^2)
f_I	Illuminated surface fraction for the PBR (no dimension)
F_0	Mean incident light flux (W/m^2)
H	Coincidence horizon (expressed in number of sampling periods)
I_Σ	Mean local available radiant light energy (W/m^2)
K_I	Half saturation constant for kinetic law (W/m^2)
q	Proportionality constant between illuminated and dark operative volumes (no dimension)
R	Radius of the cylindrical PBR (m)
r_X	Local volumetric biomass growth rate ($kg/m^3/h$)
$\langle r_X \rangle$	Mean volumetric biomass growth rate ($kg/m^3/h$)
t	Time (h)
V	Total volume of the PBR (m^3)
V_2	Dark operative volume of the PBR (m^3)
V_3	Working illuminated volume of the PBR (m^3)
x	Relative abscissa along the radius R (no dimension)
x'_3 x_3	Peculiar values of x delimiting the volume V_3 ($x'_3 \leq x_3$)
μ_M	Maximum specific growth rate (h^{-1})
λ	Reference trajectory dynamic (no dimension)

Remark :
In the present TN, the relative radius, x, is the complementary to 1 of the optical thickness, Z, defined in TN45.1 :

$$x + Z = 1$$

1. INTRODUCTION

Based on the new first principles model elaborated by J.F. Cornet of LGCB (in TN 45.1), a Predictive Functional Control is built for the compartment II (rhodospirillum rubrum photobioreactor) of the MELISSA loop.
The study is done on a Simulink® simulator (figure 1) before transplantation on the real process.

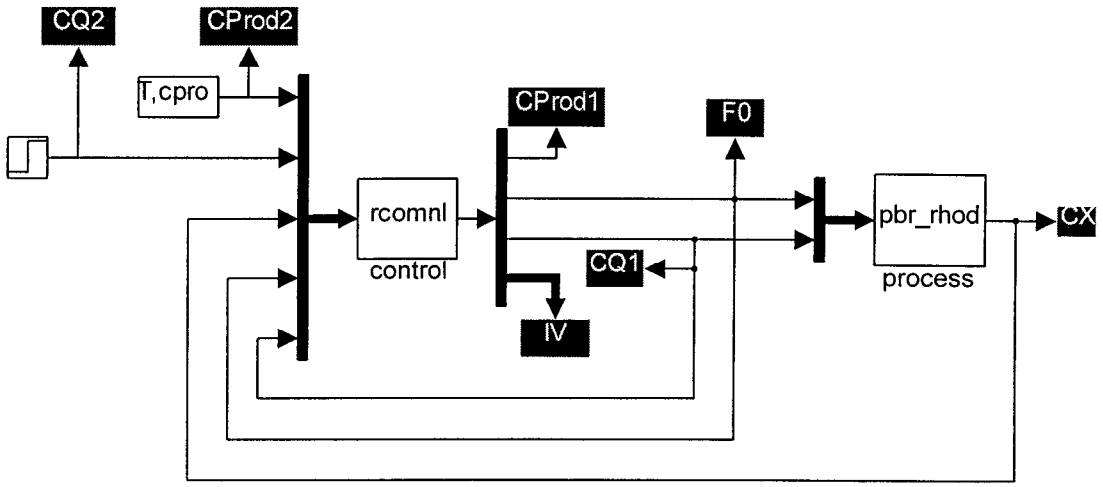

Figure 1 : Controlled compartment II

The process is defined by the following differential equation :

$$\frac{dC_{Xp}}{dt} = \langle r_{Xp} \rangle - \frac{Q_p}{V_p} \cdot C_{Xp} \qquad (1)$$

with

C_{Xp} : total biomass concentration
$\langle r_{Xp} \rangle$: mean volumetric biomass growth rate
Q_p : flow rate
V_p : volume of the photobioreactor

The index *p* means that the variables are associated to the process.

This differential equation is integrated by the rk45 (Runge-Kutta fourth and fifth order formulas) of Matlab®.

3. DESCRIPTION OF THE LEVEL 1 CONTROL

3.1. Internal model

Similarly to the simulated process, the internal model of the control is defined by the differential equation :

$$\frac{dC_X}{dt} = \langle r_X \rangle - \frac{Q}{V} \cdot C_X \qquad (2)$$

with

C_X : total biomass concentration
$\langle r_X \rangle$: mean volumetric biomass growth rate
Q : flow rate
V : volume of the photobioreactor

This equation has to integrated on the coincidence horizon, H, of the PFC, i.e. from the current instant n to the instant n+H. As H is equal to a few sampling periods (typically, less than 10), the Euler method is quite convenient for the integration.

According to the first principles model, the mean volumetric biomass growth rate of the working illuminated volume V_3, $\langle r_{X3} \rangle$, is :

$$\langle r_{X3} \rangle = 2 \cdot \mu_M \cdot C_X \cdot J \qquad (3)$$

with $J = \int_{Y3} g \cdot dx$ $\qquad Y_3 = [0 \quad x_3'] \cup [x_3 \quad 1]$

and $g = \frac{I_\Sigma}{K_I + I_\Sigma} \cdot x$

As it is shown in annex 2, the function g is monotonic on the integration interval. So J is computed by the trapezium method.

The computation of $\langle r_X \rangle$ involves the two characteristic volumes V2 and V3, V2 being the dark operative intermediate zone and V3 the working illuminated one. The computation of these volumes V2 and V3, defined in TN45.1 for rectangular photobioreactor, is adapted in the present control for cylindrical photobioreactor (annexes 3 and 4) :

$$\langle r_x \rangle = k \cdot \langle r_{x3} \rangle \qquad (4)$$

with k equal to the minimum of the 2 terms :

$$\left\{ q, \quad \frac{1}{1 + x'^2_3 - x^2_3} \right\}$$

where, by convention, a root x'_3 or x_3 that does not exist, is set to 0 .

3.2 Static bias cancellation

The method used to cancel the static bias attached to the present non linear PFC is identical to the one developed for the Spirulina photobioreactor and is detailed in TN38.2 .

4. ROBUSTNESS STUDY

4.1 Tuning parameters

The tuning parameters are those of a classical PFC :

- H: the coincidence horizon,
- λ : the dynamic of the reference trajectory.

4.2 Simulation

Tests in simulation allow to check the control code and are the only way to evaluate the robustness of the control. In the following tests, the tuning parameters have the following values :

- dynamic of the reference trajectory : $\lambda = 0.90$;
- coincidence horizon : H = 5 control periods.

The control period is : dt = 0.5 hour.
The illuminated surface fraction, f_l is set to 0.6 and the maximum specific growth rate, μ_M, is fixed to 0.15 h^{-1} (case of acetate substrate).

All the tests have the same pattern :

- At the beginning of the simulation, the process works in steady state and the biomass production set point is equal to 0.18 g/h (this set point, which is supposed to be fixed by the operator or by the level 2 of the hierarchical control, is called set point -level2 in the figures);
- At time t = 10 h (simulated time), the set point is increased to 0.25 g/h, which is higher than the maximum production allowed by the flow rate (0.14 l/h) and the maximum biomass concentration (1.5 g/l). So the control computes the set point - level1 : 0.231 g/h;
- Then at time 90 h, the set point is decreased to 0.18 g/h;
- All along the simulation, the level2 flow rate set point is fixed to 0.14 l/h (dilution = 0.02 h^{-1}) .

Test 1 (figure 2) :
The internal model of the control matches the simulated process.

At the increasing set point (t = 10 h), the control moves the two actions :
- the light intensity (F_0_ctrl) is set to its maximum constraint : 400 W/m^2 . Due to this constraint, the dynamic is limited and the set point is reached in about 20 h ;
- the water flow rate is set to its maximum value too : 0.154 l/h .

When the measured production reaches its set point (at about t = 30 h), the light intensity, F_0_ctrl, decreases smoothly to its steady state value (85 W/m^2) and causes no overshoot of the set point.

At the decreasing set point (t = 90 h), the controller moves the 2 actions as previously :
- the light intensity (F_0_ctrl) is set to its minimum value : 10 W/m^2 and the dynamic of the closed loop is due to the dilution phenomenon. The set point is reached in 20 h ;
- the water flow rate is set to its level 2 set point : 0.14 l/h .

When the measured production reaches its set point (at about 120 h), the light intensity, F_0_ctrl, increases smoothly to its steady state value (60 W/m^2) and causes no overshoot of the set point.

The maximum constraint on the biomass concentration (set to 1.5 g/l in these tests) is quite respected.

Test 2 (figure 3) :
The internal model of the control does not match the simulated process : the light intensity received by the process is assumed to be 50 W/m^2 lower than the action F_0_ctrl computed by the control.

In that case, the behaviour of the closed loop system is nearly unchanged, except that the transient duration of the increasing step is a little bit longer (30 h instead of 20 h).

Test 3 (figure 4) :
Another mismatch is simulated : the illuminated surface fraction of the process, $f_{l\_process}$, is supposed to be 75 % of the illuminated surface fraction of the internal model, f_{l_ctrl}. In comparison with the matched control, this is equivalent to a decreasing of 25 % of the biomass growth rate.

In that case too, globally the behaviour of the closed loop system is similar to the one of the matched control, except that the transient duration of the first step is longer (55 h instead of 20 h) due to the reduced biomass growth rate of the simulated process. In that case of mismatch, the dilution phenomenon is not affected; so the transient behaviour of second step is unchanged.

Test 4 (figure 5) :
This test is a simulation of bad calibration of a sensor : the flow rate of the process is assumed to be 80 % of the flow rate measured and given as input data of the internal model.

In that case too, globally the behaviour of the closed loop system is identical to the one of the matched control, except that the transient duration of the first step is a little bit shorter (because of overestimation of the production) and that the one of the last step is longer (because of a smaller dilution).

Test 5 (figure 6) :
In this test, the biomass concentration of the process is assumed to be 80 % of the measured biomass concentration.

In that case too, globally the behaviour of the closed loop system is identical to the one of the matched control.

Conclusion of these tests :
The robustness of the control is quite correct : it is able to counter all the tested mismatches without oscillations nor overshoot.

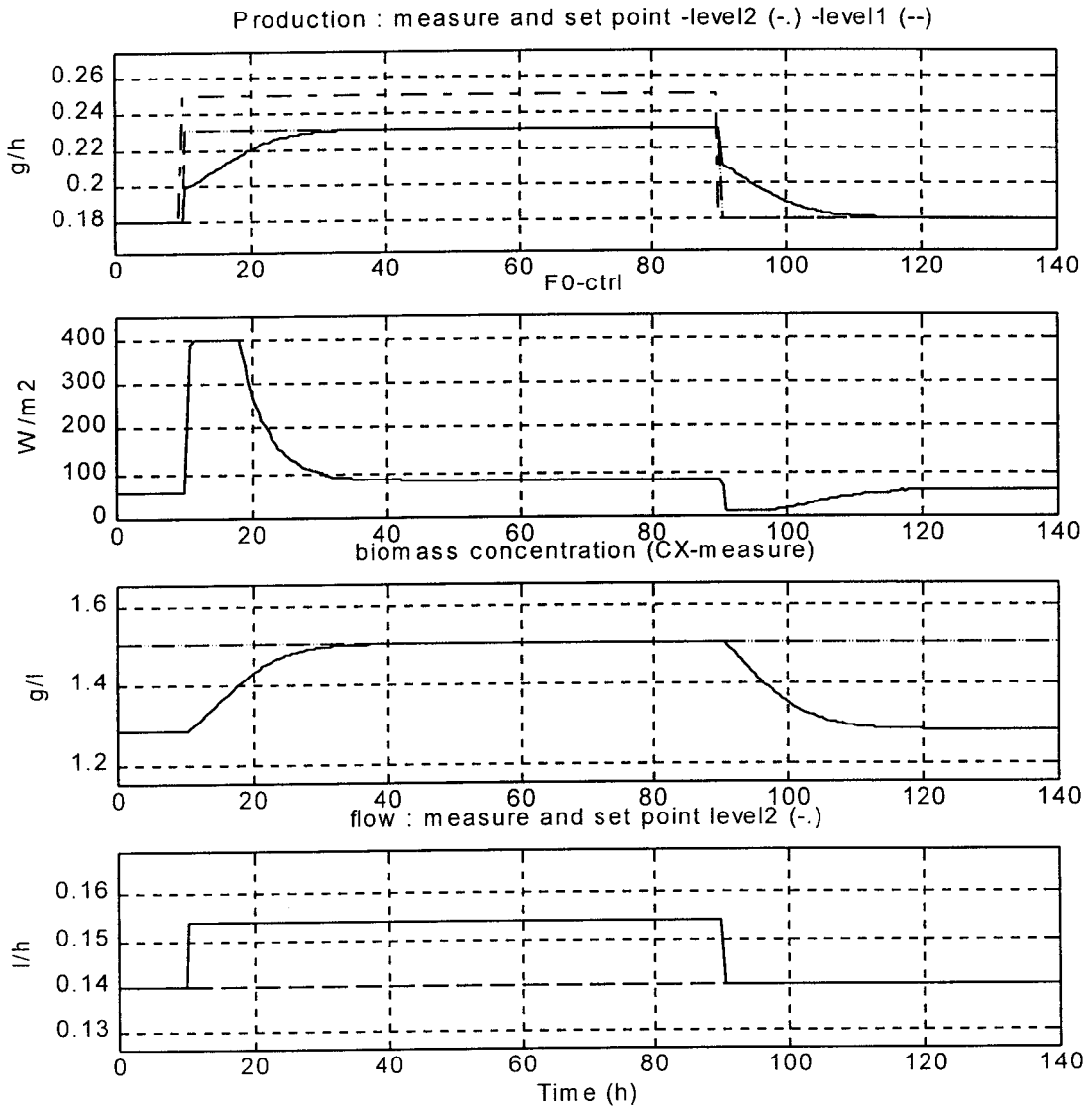

Figure 2 : Matched control

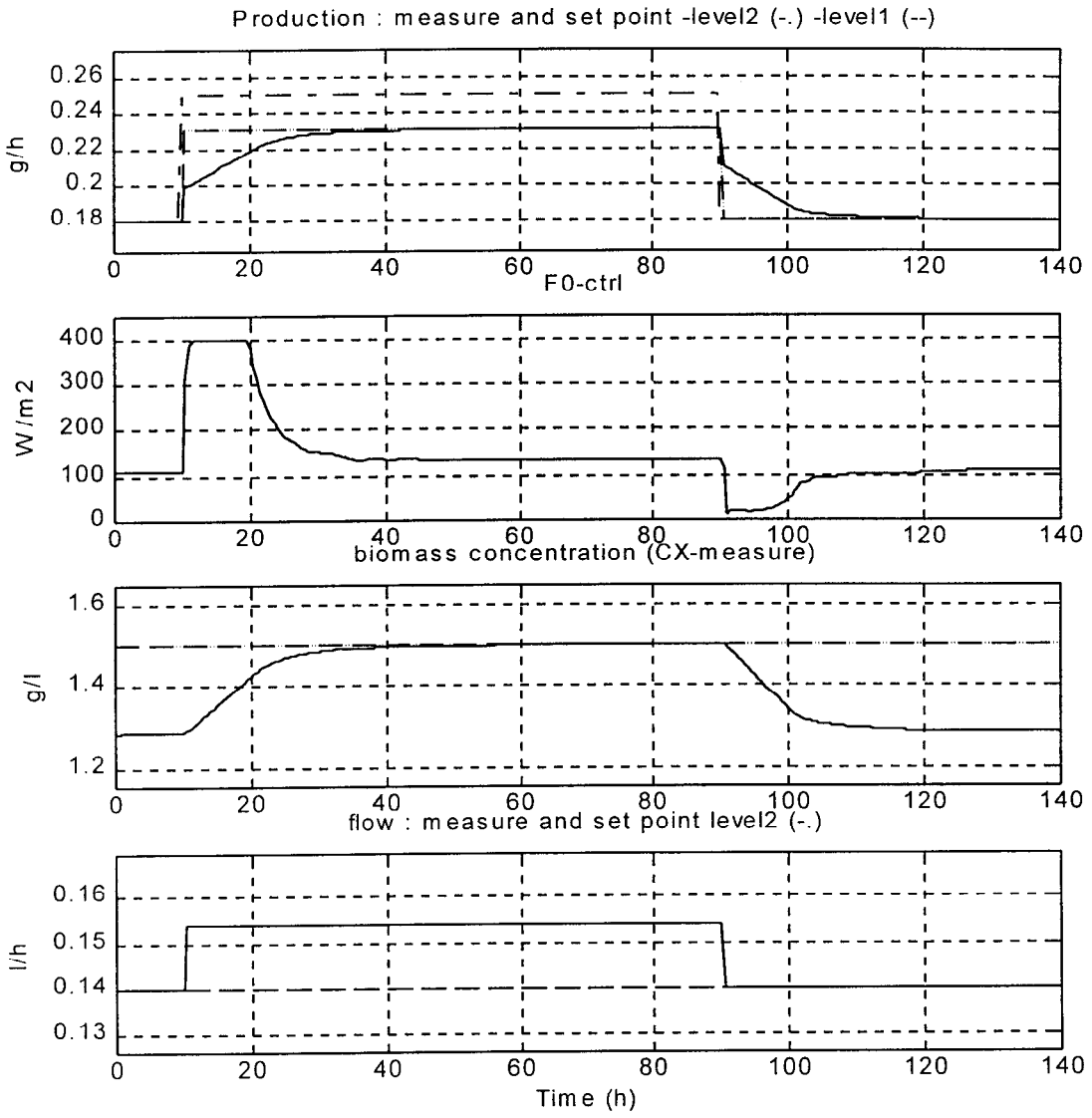

Figure 3 : Mismatched control : F_0_process = F_0_ctrl - 50

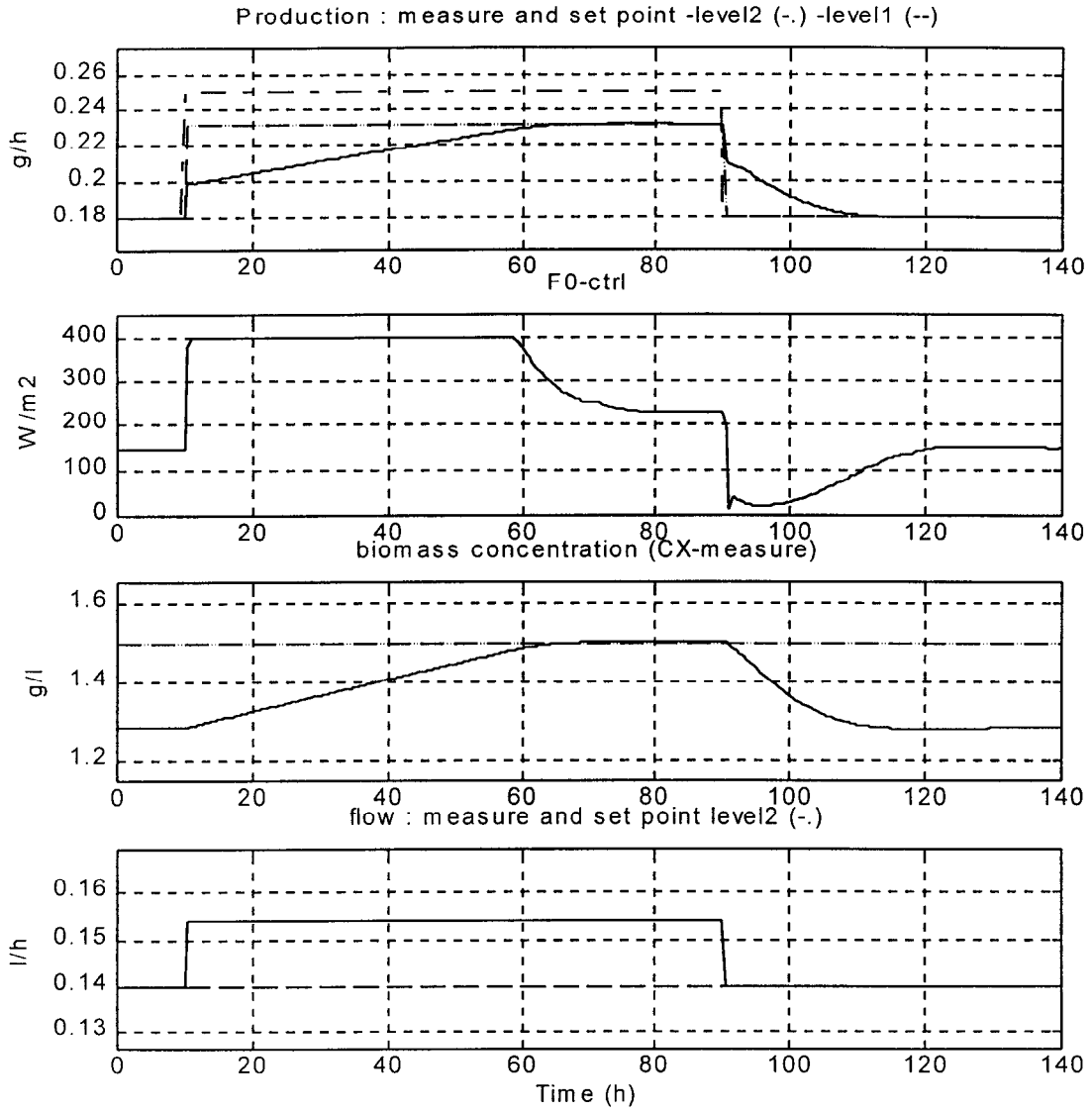

Figure 4 : Mismatched control : f_{l}_process = f_{l}_ctrl * 0.75

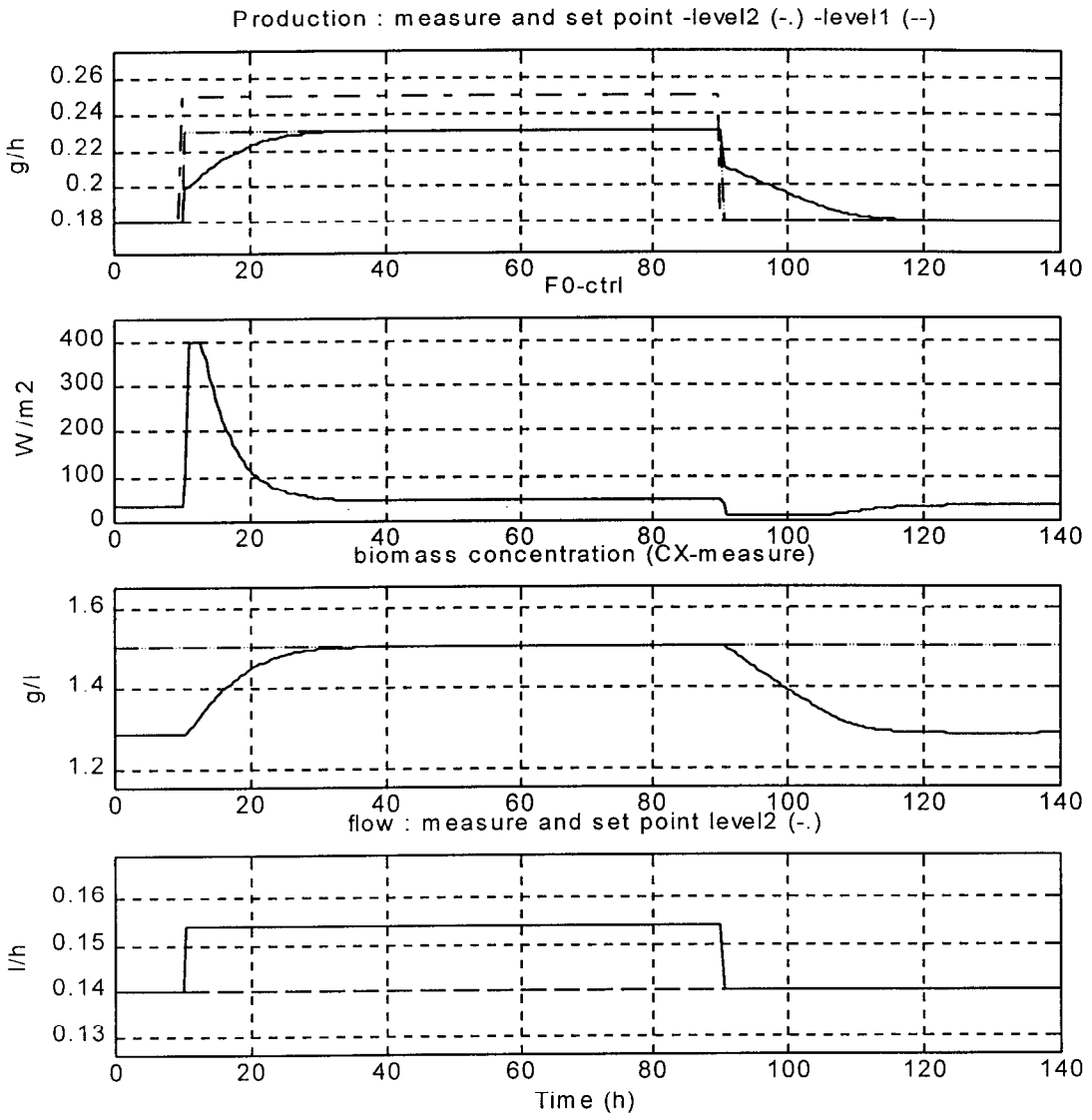

Figure 5 : Mismatched control : flow_process = flow_measure * 0.8

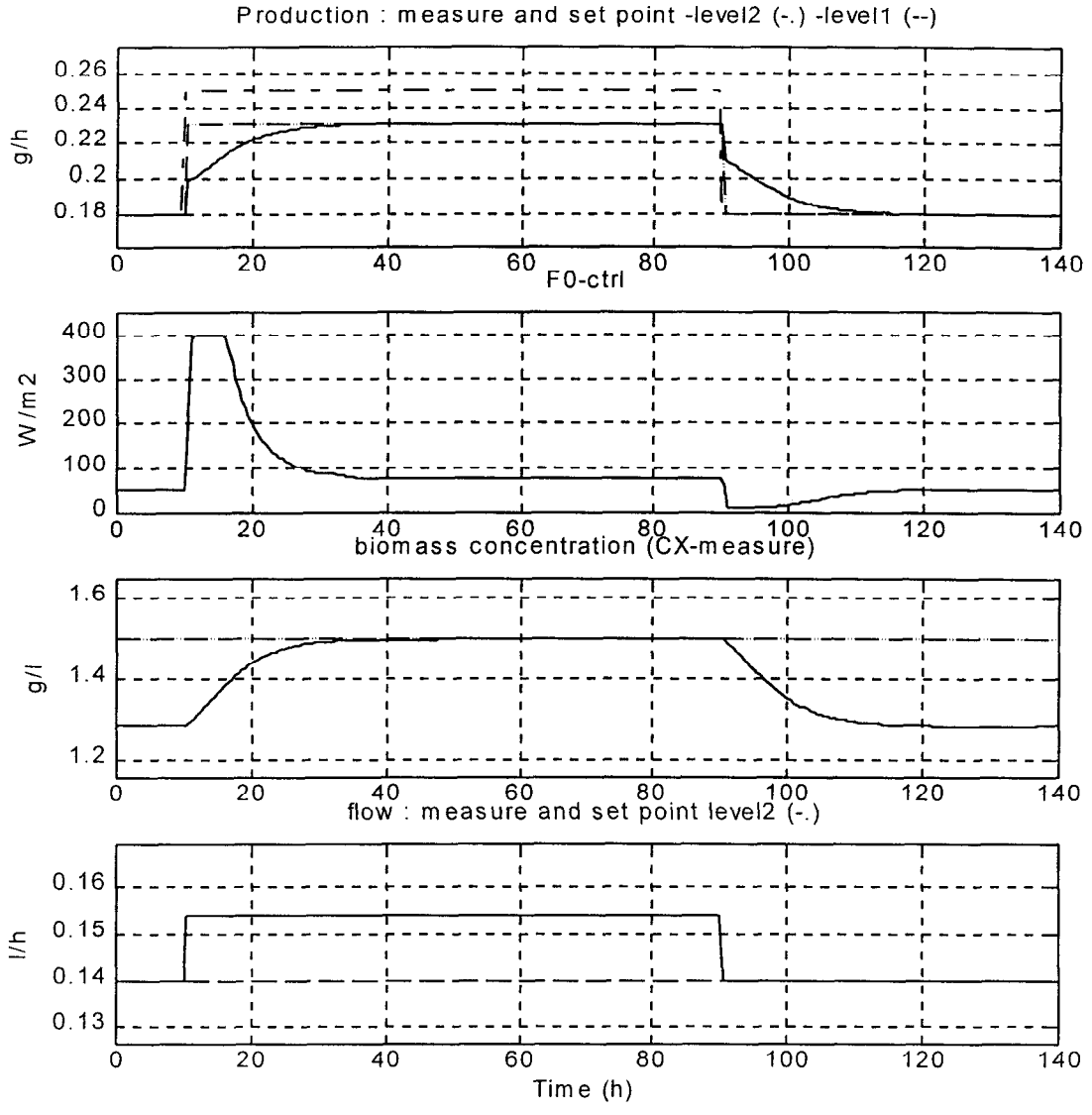

Figure 6 : Mismatched control : CX_process = CX_measure * 0.8

5. CONCLUSION

The control is tuned in simulation with the following control parameters :

- dynamic of the reference trajectory : $\lambda = 0.90$;
- coincidence horizon : $H = 5$ control periods;

and with the maximum specific growth rate specific to the acetate substrate.
This last parameter of the first principles model is an input argument of the control software *lrpc* and can be modified by the control's user according to the substrate.

Its robustness is quite satisfactory.

Apart from this control study (in annex A2), it can be noted that :

- when C_X goes down 0.4 g/l, the mean growth rate $\langle r_X \rangle$ decreases quickly;
- when C_X goes over about 3.0 g/l, $\langle r_X \rangle$ becomes constant (all the incident energy is consumed by the biomass);
- beyond 200 W/m^2 , the increasing of $\langle r_X \rangle_{max}$ with F_0 is expensive in terms of energy : a 20 % increase of $\langle r_X \rangle_{max}$ needs a 100 % increase of F_0.

So, on a real reactor, the practical domain of C_X should be :

$$C_X \in [0.4 \quad 3.0] \quad g/l$$

and the practical domain of F_0 should be :

$$F_0 \in [10 \quad 200] \quad W/m^2$$

6. REFERENCES

CORNET J.F., DUSSAP C.G., GROS J.B. "Modelling of physical limitations in photobioreactors. Adaptation of the light energy transfer model to cylindrical geometry" ESA contract PRF 130820, 1993, TN 19.1 .

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ANNEX 1

Computation of the manipulated variable by a scenario strategy

1 - Principle of the method

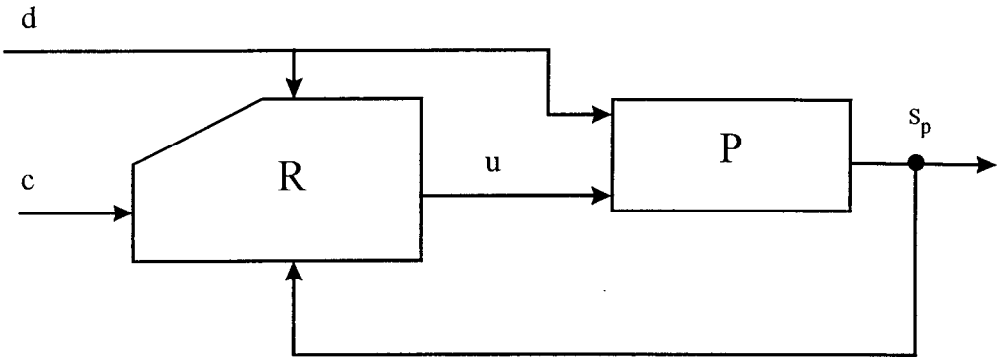

Figure A1.1 : controlled system

P : non linear process
R : controller

u : manipulated variable (F_0)
s_p : process output (production)
c : set point
d : disturbance

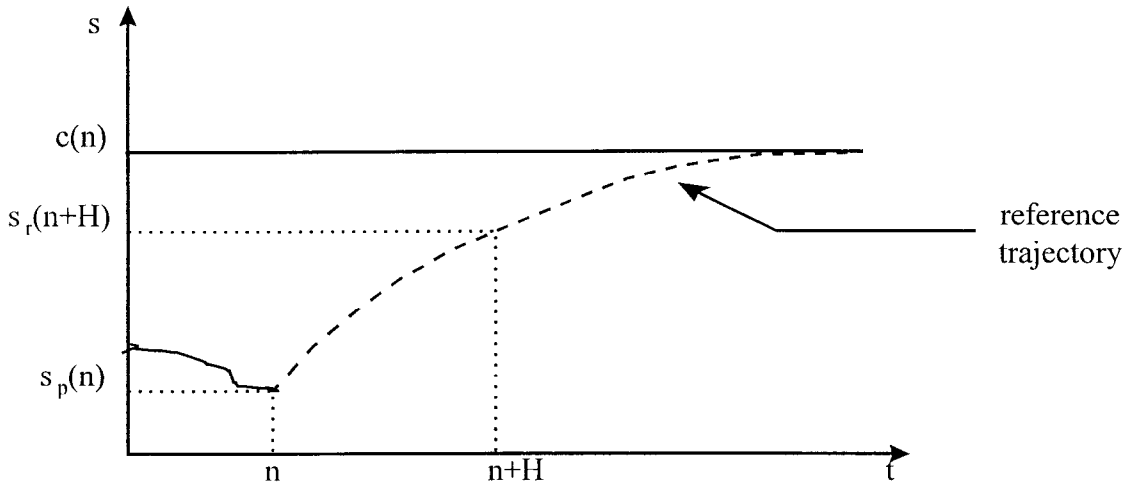

Figure A1.2 : First order reference trajectory

n : current instant
H : coincidence horizon
s_r : reference trajectory
s_m : model output

The method is based on the PFC principles applied to a non linear process.

The reference trajectory is a first order trajectory starting at $s_p(n)$ and rallying the set point c(n) assumed constant in the future :

$$s_r(n+H) = \alpha^H . s_p(n) + (1 - \alpha^H) . c(n) \qquad (A1.1)$$

with $\alpha = e^{-\frac{T}{\tau}}$ τ = time constant of the first order

At each instant n, the manipulated variable, u(n), is computed such that, when it is assumed constant on the coincidence horizon (from n to n+H) :

$$s_p(n+H) = s_r(n+H) \qquad (A1.2)$$

The estimation of the output process is done by means of the first principles model :

$$s_p(n+H) = s_m(n+H) \qquad (A1.3)$$

So, from these 3 relations, the model output at n+H is deduced :

$$s_m(n+H) = \alpha^H . s_p(n) + (1 - \alpha^H) . c(n) \qquad (A1.4)$$

All the variables of the right hand side term of (A1.4) are known and the problem, now, is to express the relation between u(n) and $s_m(n+H)$.

As this expression is not easy, the value of the MV, u(n), is approached by means of an iterative computation.

2 - Iterative computation of the MV

It is based on the fact that s_m is a monotonic ascending function of u when the disturbance d is constant :

$$s_m = f(u) \qquad (A1.5)$$

with f monotonic

Given u_0, the value of u at previous instant n-1 (u_0 exists always even at the initialization of the control) and δu, an appropriate signed variation of u, constant and not null.

The first step of the iterative loop is the computation of s_{m1} and s_{m2}, values of s_m at instant n+H for 2 values of the manipulated variable, u_0 and $u_0 + \delta u$:

$s_{m1} = f(u_0)$
$s_{m2} = f(u_0 + \delta u)$

Then, an approached value, u', is computed by the linear relation :

$$\frac{u' - u_0}{\delta u} = \frac{s_m(n+H) - s_{m1}}{s_{m2} - s_{m1}} \qquad \text{(A1.6)}$$

If the relative distance between the approached solution and the target :

$$\delta y = \frac{f(u') - s_m(n+H)}{s_m(n+H)} \qquad \text{(A1.7)}$$

is small enough, the iterative loop is stopped ; otherwise another loop is begun again.

ANNEX 2

Main characteristics of the first principles model

1 - Recall of the model

According to the first principles model built by J.F. Cornet for cylindrical and radially illuminated photobioreactors, the expression of the mean local available radiant light energy, I_Σ, is :

$$I_\Sigma = F_0 \cdot \frac{R}{r} \cdot \frac{2\,\mathrm{ch}\left[\delta \cdot \frac{r}{R}\right]}{\mathrm{ch}\,\delta + \alpha \cdot \mathrm{sh}\,\delta} \qquad \text{(A2.1)}$$

with :

R : radius of the photobioreactor

r : locates the local cylindrical surface at distance r from the axis of the photobioreactor

F_0 : mean incident light flux

α and δ : defined in TN 45.1

According to this model too, the local growth rate, in the working illuminated volume V_3, is :

$$r_{X3} = \mu_M \cdot C_X \cdot \frac{I_\Sigma}{K_J + I_\Sigma} \qquad \text{(A2.2)}$$

and the mean volumetric growth rate is :

$$\langle r_{X3} \rangle = \frac{1}{V} \cdot \int_{V3} r_{X3} \cdot dV \qquad \text{(A2.3)}$$

Given the relative abscissa $x = \frac{r}{R}$,

the relation (A2.3) becomes :

$$\langle r_{X3} \rangle = 2 \cdot \mu_M \cdot C_X \cdot J \qquad \text{(A2.4)}$$

with $J = \int_{Y3} g \, . \, dx$ $\qquad Y_3 = [0 \quad x_3'] \cup [x_3 \quad 1]$

and $g = \dfrac{I_\Sigma}{K_J + I_\Sigma} \, . \, x$

2 - Study of the function g

The derivative of g versus x is :

$$\dot{g} = \frac{N}{D}$$

with :

$D = (K_J + I)^2 \qquad > 0$

$N = K_J \, . \, \dot{I} \, . \, x + K_J \, . \, I + I^2$

Now given $y = I \, . \, x$

its derivative is $\dot{y} = \dot{I} \, x + I$

then $N = K_J \, . \, \dot{y} + I^2$

Now $y = \dfrac{2 \, . \, F_0}{ch \, \delta + \alpha \, sh \, \delta} \, . \, ch(\delta x)$

then $\dot{y}$ is positive as $ch(\delta x)$ is ascending on $[0 \quad +\infty[$.

Therefore $\dot{g}$ is positive and g is monotonic ascending on $[0 \quad 1]$.

Noticing that, when x tends to 0, I_Σ and g tend to $+\infty$ and 0, respectively, the typical aspect of g is plotted in figure A2.1.

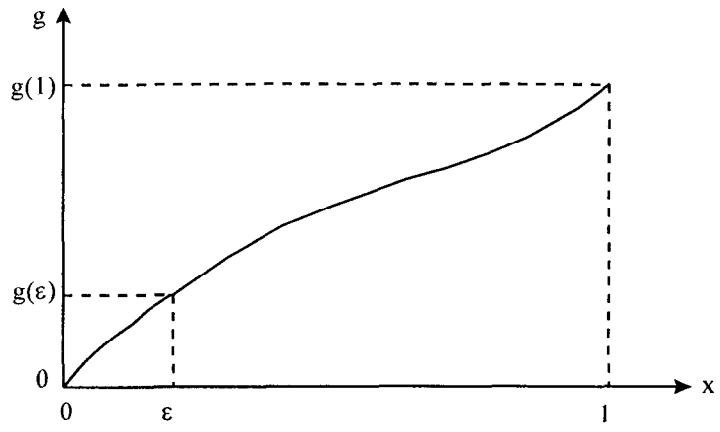

Figure A2.1 : General aspect of g on [0 1]

3 - Illustrations of I_Σ and g

In order to illustrate the aspects of I_Σ and g, 4 examples are given in which the incident light flux, F_0, is increasing from 10 to 400 W/m^2 while the biomass concentration, C_x, is decreasing from 2 to 0,1 kg/m^3 (table A2.1).

Example	F_0 (W/m^2)	C_x (kg/m^3)
1	10	2
2	100	0.67
3	200	0.28
4	400	0.1

Table A2.1 : F_0 and C_x of the illustrations

These illustrations are plotted in the figure A2.2 on 2 columns of graphs : the I_Σ curves are plotted in the left hand side one and the g curves in the right hand side one.

Each row of graphs is associated to one of the 4 examples (example 1 to example 4, from top to bottom).

The 2 roots, x'_3 and x_3 , of

$$I_\Sigma = \mathcal{E}_J$$

are given into brackets : [x'_3 x_3] (with $x'_3 \leq x_3$) in the graphs of the function I_Σ. When a root does not exist, it is replaced by 0.

The values of J and r_{x3} are also given in the right hand side graphs.

They illustrate that g is monotonic increasing on [0 1], as previously demonstrated.

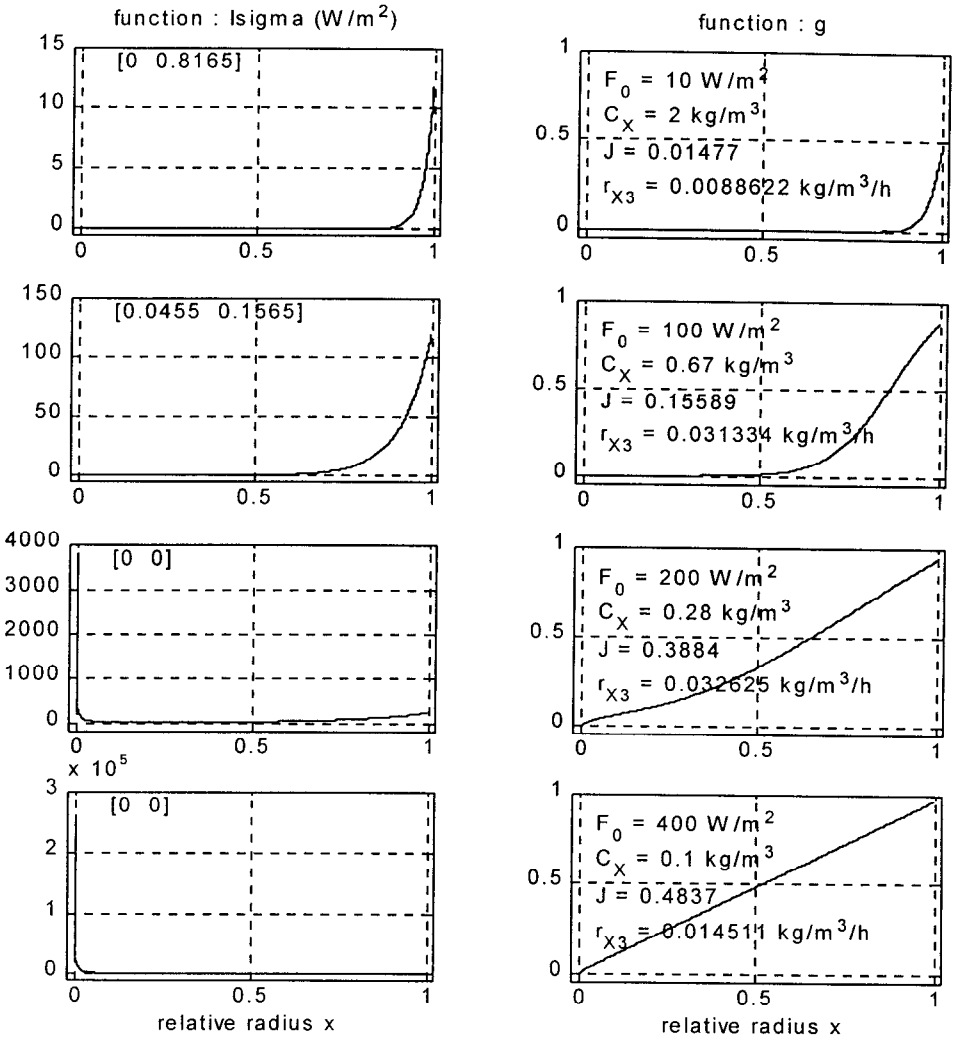

Figure A2.2 : Illustrations of I_Σ and g

4 - Domains of F_0 and C_X

The figure A2.3 gathers the variation of J and $\langle r_X \rangle$ in function of C_X for a few values of F_0 : the J curves are plotted in the left hand side column and the $\langle r_X \rangle$ curves in the right hand side one.
The variation of the maximum of $\langle r_X \rangle$ versus F_0 is plotted in figure A2.3.

This short numerical study shows that :

- when C_X goes down 0.4 g/l, the mean growth rate $\langle r_X \rangle$ decreases quickly;
- when C_X goes over about 3.0 g/l, $\langle r_X \rangle$ becomes constant (all the incident energy is consumed by the biomass);
- beyond 200 W/m^2 , the increasing of $\langle r_X \rangle_{max}$ with F_0 is expensive in terms of energy : a 20 % increase of $\langle r_X \rangle_{max}$ needs a 100 % increase of F_0.

So, on a real reactor, the practical domain of C_X should be :

$$C_X \quad \in \quad [0.4 \quad 3.0] \quad g/l \qquad \text{(A2.5)}$$

and the practical domain of F_0 should be :

$$F_0 \quad \in \quad [10 \quad 200] \quad W/m^2 \qquad \text{(A2.6)}$$

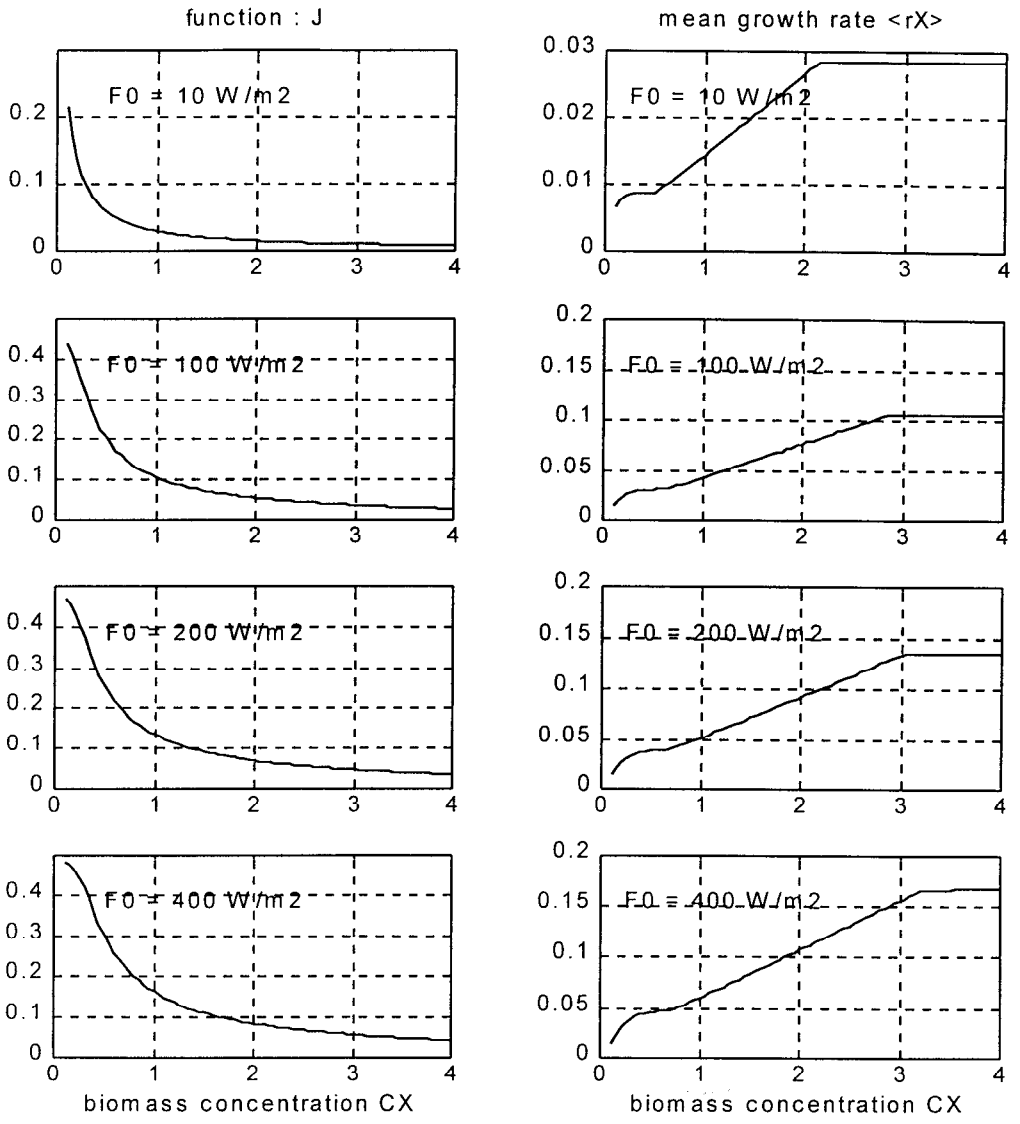

Figure A2.3 : Functions J and $\langle r_X \rangle$

$\langle r_X \rangle$ in kg/m^3/h

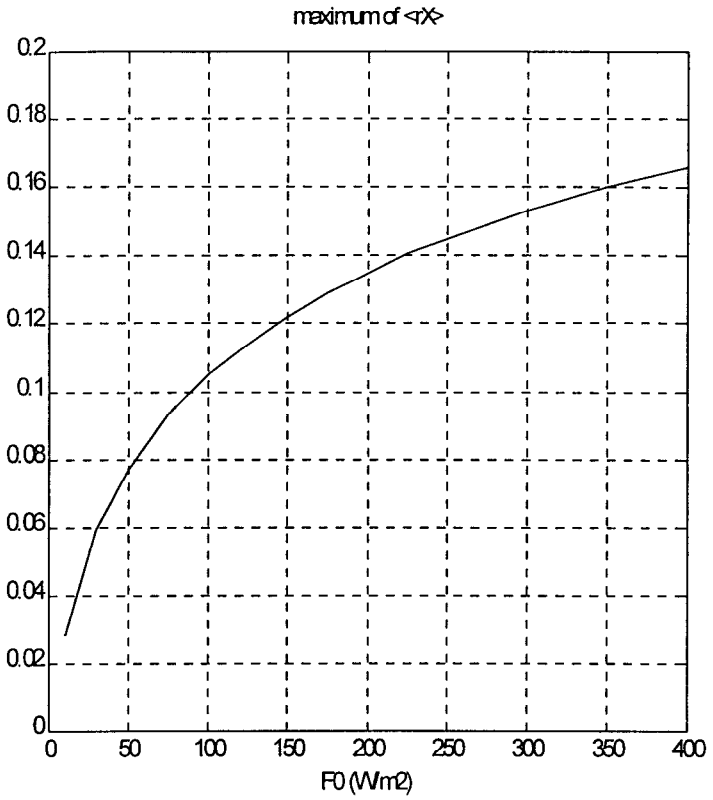

Figure A2.4 : Evolution of $\langle r_X \rangle_{max}$ versus F_0

$\langle r_X \rangle_{max}$ in kg/m^3/h

ANNEX 3

Computation method of the limits x'_3 and x_3 of the working illuminated volume

From the data processing point of view, it is common to associate, in one hand, the determination of x_3 and, in the other hand, the integration of g by the trapezium method : these 2 operations can be stated in a single iterative loop and in a very simple way.

The only problem is to determine the number of intervals which gives a good precision on $\langle r_{X3} \rangle$. From (A2.4), the relative error on $\langle r_{X3} \rangle$ is the same as on J.

1 - Relative error on J

The accuracy on the relative abscissa x_3, delimiting the outer part of the working illuminated volume, depends directly on the number of integration steps.

Given x_0 the exact value of x_3 so that :

$$I_\Sigma(x_0) = \mathcal{E}_J \qquad \text{(mean efficient intensity)}$$

Given x_n and x_{n+1}, the values of x at steps n and n+1, around x_0 (figure A3.1) such that
$I_\Sigma(x_n) \leq \mathcal{E}_J$ and
$I_\Sigma(x_{n+1}) > \mathcal{E}_J$

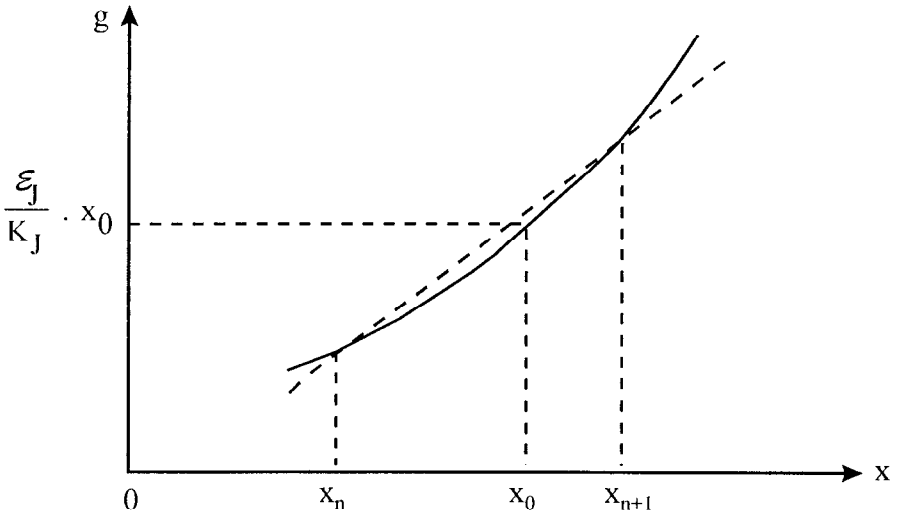

Figure A3.1 : Function g around x_3

As $\mathcal{E}_J$ is negligible in comparison with K_J :

$$g(x_0) \simeq \frac{\varepsilon_J}{K_J} x_0$$

Given : $J_e = \int_{x_0}^{1} g \, . \, dx$ (exact value)

and $J_a = \int_{x_{n+1}}^{1} g \, . \, dx$ (approximate value)

The absolute error on J is :

$$\Delta J = J_e - J_a = \int_{x_0}^{x_{n+1}} g \, . \, dx$$

$$\Rightarrow \quad \Delta J \simeq \frac{\varepsilon_J}{K_J} \, . \, x_0 (x_{n+1} - x_0) \qquad \text{(A3.1)}$$

Given n_i the number of integration steps.

Now $x_0 \leq 1$ and $x_{n+1} - x_0 < \frac{1}{n_i}$

Then $\Delta J < \frac{\varepsilon_J}{K_J \, . \, n_i}$ (A3.2)

Assuming that the same error is done with the other relative abscissa, x'_3, if any, the total absolute error on J is :

$$\Delta J < \frac{2 \, . \, \varepsilon_J}{K_J \, . \, n_i} \qquad \text{(A3.3)}$$

So, the absolute error on J, ΔJ, is maximized by a constant term depending on n_i :

$$A = \frac{2 \, . \, \varepsilon_J}{K_J \, . \, n_i}$$

Assuming true the relations (A2.5) and (A2.6) :

$$C_X \quad \in \quad [0.4 \quad 3.0] \quad g/l$$

$$F_0 \in [10 \quad 200] \quad W/m^2$$

the minimum value of J is about 10^{-2} (top left graph of figure A2.3) and the relative error on J is maximized in the table A3.1.

n_i	A	max relative error on J
100	$1.3 \ 10^{-5}$	10^{-3}
1000	$1.3 \ 10^{-6}$	10^{-4}

Table A3.1 : Max relative error on J versus the number of integration steps n_i

2 - Number of integration steps

As a relative error on J equal to 10^{-3} is quite convenient, the number of integration steps n_i is set to :

$$n_i = 100 \qquad \text{(A3.4)}$$

ANNEX 4

Expression of the mean volumetric growth rate Adaptation to cylindrical reactor

From the following equations (6) and (9) of TN 45.1 :

$$\langle r_X \rangle = \frac{1}{V}\left[\int_{V_2} r_{X2} \,.\, dV + \int_{V_3} r_{X3} \,.\, dV\right] \qquad \text{(6 of TN 45.1)}$$

$$\frac{1}{V_2}\int_{V_2} r_{X2} \,.\, dV = \frac{1}{V_3}\int_{V_3} r_{X3} \,.\, dV \qquad \text{(9 of TN 45.1)}$$

it comes :

$$\langle r_X \rangle = k \,.\, \langle r_{X3} \rangle \qquad \text{(A4.1)}$$

with :

$$k = \frac{V_2 + V_3}{V_3}$$

In the TN 45.1, where the reactor is rectangular with an infinite length L, the volume V_2 exists and :

$$k = q = 3.2 \pm 0.2$$

In the case of a finite cylindrical reactor, when V_2 exists, the sum of V_2 and V_3 is limited to the total volume V :

$$V_2 + V_3 \leq V$$

and k is limited by the relation :

$$k \leq \frac{1}{1 + x'^2_3 - x^2_3} \qquad \text{(A4.2)}$$

where x'_3 and x_3 are the relative abscissa along the radius of the reactor delimiting the working illuminated volume V_3 defined in TN 45.1.

So the expression of r_x, adapted to the cylindrical reactor, is :

$$\langle r_X \rangle = k \,.\, \langle r_{X3} \rangle \qquad \text{(A4.3)}$$

with k equal to the minimum of the 2 terms :

$$\left\{ q, \quad \frac{1}{1 + x'^2_3 - x^2_3} \right\}$$

where, by convention, a root x'_3 or x_3, that does not exist, is set to 0 .

Remark :

When the equation :

$$I_\Sigma = \mathcal{E}_J$$

has no root x'_3 nor x_3,

then

$V_3 = V$ and V_2 does not exist .

So $k = \dfrac{V_2 + V_3}{V_3}$ is equal to 1

as obtained from $\dfrac{1}{1 + x'^2_3 - x^2_3}$ where x'_3 and x_3 are null.

ANNEX 5

Source file of the control : lrpc.c

```
/*
   LRPC.C   Light Rhodobacter Production Control
   Version 1.0

   AUTHOR : LECLERCQ JJ (ADERSA)
   October 1999

------------------------------------------------------------------------
   Function:
      Non linear PFC control of Rhodobacter production by light
   Synopsis:
      LRPC(PROD_SP2,CX,QE_SP2,F0,QE_MES,SM_SUP,CONS_SUP,
           VOL,FI,MUM,DT,LAMBDA,INIT,VAR_OUT,TRACE)
      (v) : numerical value
      (p) : pointer
      PROD_SP2 (v):level2 production set point (g/h)
      CX       (v):biomass concentration (g/l)
      QE_SP2   (v):level2 flow rate set point (l/h)
      F0       (p):light intensity : measured or computed by the control (W/m2)
                   . input argument : measured value of F0
                   . output argument : computed by the control
      QE_MES   (v):measure of flow rate (l/h)
      SM_SUP   (p):production model output computed by the supervisor (g/h)
                   . input argument : value at previous moment
                   . output argument : value at present moment
      CONS_SUP (p):production set point computed by the supervisor (g/h)
                   . input argument : value at previous moment
                   . output argument : value at present moment
      VOL      (v):volume of the reactor (l)
      FI       (v):illuminated surface fraction (no dimension)
      MUM      (v):maximum specific growth rate for acetate     (1/h)
      DT       (v):control period (h)
      LAMBDA   (v):dynamic of the reference trajectory (dimension less)
      INIT     (p):initialisation flag (when equal to 0)
                   put to 1 by this programme
      VAR_OUT[0]   :level 1 production set point (g/h)
      VAR_OUT[1]   :level 1 flow rate set point (l/h)
      VAR_OUT[2]   :derivative of the model biomass concentration (g/l/h)
                    (to be compared to the derivative of the process conc.)
      TRACE[50]    :array of internal variables to check the control
   Description:
      . Computation of level 1 production and flow rate set points, VAR_OUT[0]
      and VAR_OUT[1], functions of level 2 set points, PROD_SP2 and QE_SP2.
      . Computation of the Manipulated Variable at current instant n, F0(n),
      function of level 1 production and flow rate set points,
      of measured flow rate, QE_MES, and
      of light intensity measured at previous time, F0(n-1).
*/

#include <math.h>

/* prototype */
void lrpc( );
double mod_rhod( );
double dercx( );
double signe( );
double min( );
double max( );
```

```
#ifndef ADERSA
void lrpc(double prod_sp2, double cx, double qe_sp2, double *F0, double qe_mes,
          double *sm_sup, double *cons_sup, double vol, double fI, double muM,
          double dt, double lambda, double *init, double var_out[3],
          double trace[50])
#else
void lrpc(prod_sp2, cx, qe_sp2, F0, qe_mes,
          sm_sup, cons_sup, vol, fI, muM, dt, lambda, init, var_out, trace)
double   prod_sp2, cx, qe_sp2, *F0, qe_mes,
         *sm_sup, *cons_sup, vol, fI, muM, dt, lambda, *init, var_out[3],
         trace[50];
#endif


{
   double  nhc=5.;          /* coincidence point (in dt) */
   double  dF0=20.;         /* radiant flux increment (in W/m2)  */
   double  F0_min=10.;      /* min constraint on F0 (in W/m2) */
   double  F0_max=400.;     /* max constraint on F0 (in W/m2) */
   double  dq=.1;           /* flow variation (dimensionless) */
   double  cx_min=0.5;      /* min constraint on CX (in g/l) */
   double  cx_max=1.5;      /* max constraint on CX (in g/l) */


/* internal variables declaration */
   double   prod, dil, prod_ref, delF0;
   double   F01, F02, prod1, prod2, prod_mod;
   double   qe_max , qe_min , prod_max , prod_min ;
   double   cons_prod1 , cons_qe1;
   double   cons_prod0 , cons_prod0_min , cons_prod0_max;
   double   dprod_min , dprod_max;
   double   dcxdt, dy;
   short    cnt;

/* saving the input arguments in the array 'trace' */
   trace[0]=prod_sp2;   trace[1]=cx;    trace[2]=qe_sp2;        trace[3]=*F0;
   trace[4]=qe_mes;     trace[5]=vol;   trace[6]=fI;            trace[7]=muM;
   trace[8]=dt;         trace[9]=lambda;trace[10]=*init;

/* level 2 control */
   /* 1_ flow and production constraints */
   qe_max = qe_sp2*(1+dq);
   qe_min = qe_sp2*(1-dq);
   prod_max = qe_max*cx_max;
   prod_min = qe_min*cx_min;
   /* 2_ feasible production setpoint calculation */
   cons_prod1 = max(prod_min,min(prod_max,prod_sp2));
   /* 3_ real flow setpoint  and corresponding dilution rate */
   cons_qe1 = qe_sp2;
   if (cons_prod1/cx_max > qe_sp2 )
     cons_qe1 = min(qe_max,cons_prod1/cx_max);
   if (cons_prod1/cx_min < qe_sp2 )
     cons_qe1 = max(qe_min,cons_prod1/cx_min);

/* computation of dilution and biomass production
   with anticipation of the dilution              */
   dil = cons_qe1/vol;
   prod = cx * qe_mes;
```

```
/* PFC_supervisor to cancel static bias of the scenario method
   Internal model of this PFC : first order without delay
                                unit static gain
                                trbo = trbf of the embeded system */
   /* 1_ computation of the internal model output of this PFC */
   if (*init < 0.5)
   {
     prod1 = mod_rhod(cx,*F0,dil,vol,dt,nhc,fI,muM);
     *cons_sup = prod + (prod1 - prod) / (1.-pow(lambda,nhc));
     *sm_sup = *cons_sup;
     *init = 1.;
   }
   *sm_sup = lambda * *sm_sup + (1. - lambda) * *cons_sup;

   /* 2.1_ Computation of the output of the supervisor 'cons_prod0' */
   cons_prod0 = cons_prod1 - prod + *sm_sup;

   /* 2.2_ Transfer of the constraints 'F0_min' and 'F0_max' on 'cons_prod0' */
   dprod_max = mod_rhod(cx,F0_max,dil,vol,dt,nhc,fI,muM) - prod;
   cons_prod0_max = prod + dprod_max / (1.-pow(lambda,nhc));
   dprod_min = mod_rhod(cx,F0_min,dil,vol,dt,nhc,fI,muM) - prod;
   cons_prod0_min = prod + dprod_min / (1.-pow(lambda,nhc));
   cons_prod0 = max(cons_prod0_min,min(cons_prod0_max,cons_prod0));

/* reference trajectory */
   prod_ref = cons_prod0 - pow(lambda,nhc)*(cons_prod0 - prod);

/* Iterative computation of the MV by means of a scenario method */
   dy = 1.e30;  /* relative distance */
   cnt = 0;     /* counter of the 'while' loop */
   F01 = *F0;   /* saving 'F0' */
   *F0 = (F0_min+F0_max)/2.;  /* to pass the test 'while' at the first time */
   while ((dy > 1.e-3) && (cnt <= 3) && (*F0 > F0_min) && (*F0 < F0_max))
   {
     cnt ++;

/*   first scenario */
     prod1 = mod_rhod(cx,F01,dil,vol,dt,nhc,fI,muM);

/*   second scenario */
     delF0 = dF0*signe(prod_ref - prod1);
     F02 = F01 + delF0;
     if (F02 < 0.)
     {
       F02 = 0.;
       delF0 = F02 - F01;
     }
     prod2 = mod_rhod(cx,F02,dil,vol,dt,nhc,fI,muM);

/*   computation of the Manipulated Variable 'F0' */
     *F0 = F01 + (prod_ref - prod1)/(prod2 - prod1)*delF0;

/*   relative distance 'dy' between solution and target */
     prod1 = mod_rhod(cx,*F0,dil,vol,dt,nhc,fI,muM);
     dy = fabs((prod_ref - prod1) / prod1);

     F01 = *F0;          /* for the next 'while' loop, if any */
   }

/* constraints on F0 */
```

```
   *F0 = max(F0_min,min(F0_max,*F0));

   /* printf(" dy cnt \t%13.5e %13.5e\n", dy, (float)cnt); */

/* model of the derivative of the biomass concentration */
   dcxdt = dercx(cx,*F0,dil,fI,muM);

/* transfer of output arguments */
   *cons_sup = cons_prod0;
   var_out[0] = cons_prod1;
   var_out[1] = cons_qe1;
   var_out[2] = dcxdt;

/* saving output arguments in the array 'trace' */
   trace[11]=cons_prod1;        trace[12]=cons_qe1;     trace[13]=*cons_sup;
   trace[14]=*sm_sup;           trace[15]=*F0;          trace[16]=dcxdt;
   trace[17]=dy;

   return;
}
/* ---  MOD_RHOD  ------------------------------------------------------------
   Function:
         Integration with Euler method
   Synopsis:
         MOD_RHOD(CX,F0,DIL,VOL,DT,NHC,FI,MUM,PROD)

*/
double mod_rhod(cx,F0,dil,vol,dt,nhc,fI,muM)

double cx, F0, dil , vol , dt , nhc , fI, muM;

{
   double v, dv, vout , prod;
   int k;

   v = cx;
   for (k=1 ; k <= (int)nhc; k++)
       {
       dv = dercx(v,F0,dil,fI,muM);
       vout =v + dt *dv;
       v=vout;
       }
   prod=vout*dil*vol;
   return (prod);
}


/* ---  DERCX  ---------------------------------------------------------------

   Function:
         Computation of the derivative of the biomass concentration
   Synopsis:
         DERCX(CX,F0,DIL,FI,DVT,MUM);

*/
double dercx(cx,F0,dil,fI,muM)

double cx, F0, dil, fI, muM;

{
```

```
/* Internal variables */
   double alpha, delta;
   double a, dx, I, J, x, rX, rX3, x3, x3p, k;
   double dcxdt;
   short i, i1;
   #define Ndim 102    /* dimension of array */
   short limitx[Ndim]; /* dimension must be >= np+2 */

/* Parameters of the RHODO first principles model (LGCB TN 45.1)
   ------------------------------------------------------------ */
   double RT=.048;  /* radius of the 7 litres reactor (m)                  */
   double Ea=270;   /* mean Schuster mass absorption coefficient (m2/kg) */
   double Es=370;   /* mean Schuster mass scattering coefficient (m2/kg) */
   double KJ=15;    /* half saturation constant                  (W/m2)   */
   double EpsJ=.01; /* mean efficient intensity                  (W/m2)   */
   double q=3.2;    /* max ratio of V2/V3                                 */

/* Parameter of the integration method
   ---------------------------------- */
   short  np=100;   /* number of integration step along the radius
                       (must be lower than dimension of array 'limitx')  */

/* test of array dimension */
   if (Ndim < np+2)
     printf(" Dimension of array too short in dercx\n");

/* general parameters
   ------------------ */
   alpha=sqrt(Ea/(Ea+Es));
   delta=(Ea+Es)*cx*alpha*RT;

/* Computation of rx
   -----------------
         Integration interval : [0  1]
         This interval is divided into 'np' equal parts
         Integration : trapezium method

   Computation of Isigma and J
   Taking into account the mean efficient intensity EpsJ */
   dx = 1. / (float)np;
   J = 0.;
   i1 = 0;
   for (i = 1; i <= np; i ++)
   {
     x = 1./(2.*np) + (float)(i-1) * dx;
     I = 2.*F0*cosh(delta*x)/(x*(cosh(delta)+alpha*sinh(delta)));
     if (I > EpsJ)
     {
         J += x * I / (KJ + I);
     }
     else
     {
       i1 ++;
       limitx[i1] = i;
     }
   }
   J = J * dx;

/* Computation of the mean growth rate rX3 in volume V3 defined in TN45.1 */
```

```
    rX3 = 2. * fI * muM * cx * J;

  /* Boundary of the interval where I <= EpsJ */
    x3p = 0.;      x3 = 0.;
    if (i1 > 1)
    {
      x3p = 1./(2.*np) + (float)(limitx[1] -1) * dx;
      x3 =  1./(2.*np) + (float)(limitx[i1] -1) * dx;
      if (limitx[1] == 1)
        x3p = 0.;
    }

  /* Computation of the mean growth rate rX in total volume */
    a = 1 + x3p*x3p - x3*x3;
    if (a > 0)
    {
      k = min(q, 1/a);
    }
    else
    {
      k = q;
    }
    rX = k * rX3;

    dcxdt = -dil*cx + rX ;
    return (dcxdt);
}
```